DOI/ER/40762–028

UMPP #94–086

# Model Independent Extraction of $|V_{\rm bc}|$ Without Heavy Quark Symmetry.

T. D. Cohen and J. Milana

*Department of Physics, University of Maryland*

*College Park, Maryland 20742*

(February, 1994)

arXiv:hep-ph/9402249v2 16 Feb 1994

## Abstract

A new method to extract $|V_{\rm bc}|$ is proposed based on a sum–rule for semileptonic decays of the $B$ meson. The method relies on much weaker assumptions than previous approaches which are based on heavy–quark symmetry. This sum–rule only relies on the assumption that the virtual $c\overline{c}$ pair content of the $B$ meson can be neglected. The extraction of the CKM matrix element also requires that the sum–rule saturates in the kinematically accessible region.

PACS numbers: 12.15.Hh, 13.25.Hw, 13.85.Qk, 13.85.Ni

Typeset using REVTEX

The CKM quark mixing matrix element $V_{\rm bc}$, between bottom, $b$, and charm, $c$, quarks, is one of the fundamental parameters of the standard model. Direct measurements of this matrix element at the quark level are not possible because the quarks are confined in hadrons. Thus, to extract $V_{\rm bc}$ from experimental measurements it is necessary to understand the strong interactions well enough to relate the observables, which involve hadrons, to the underlying quark dynamics. In principle, if one knew the exact quantum state for an initial hadron containing a $b$ quark and an exact quantum final state containing a $c$ quark, one could extract the magnitude of $V_{\rm bc}$ from a weak decay from the initial "$b$" hadron to the final state.

Until recently it was believed that until reliable *ab initio* calculations were available directly from QCD via lattice simulations, one was unadvoidabley forced to rely on low energy models for the hadronic matrix elements. However, as was noted by Nussinov and Wetzel [1] and by Isgur and Wise [2], if the assumptions underlying heavy quark symmetry are valid, there is a model independent way to extract $|V_{\rm bc}|$ from exclusive semi-leptonic weak decays of the $B$ meson into a $D$ or $D^*$ meson; analogously, semi-leptonic decays of the $\Lambda_b$ into $\Lambda_c$ could be used. An alternative method to extract $V_{\rm bc}$ from semi-leptonic decays has been proposed by Bjorken, Dunietz and Tarron (BDT) [3]. The BDT method also depends on the validity of the assumptions underlying heavy quark symmetry; if valid, BDT show that inclusive $B$ meson or $\Lambda_b$ semi-leptonic decays into charmed states at fixed three momentum satisfy a sum-rule. In fact, this sum rule basically just reproduces the spectator model, and hence by experimentally measuring the semi-leptonic partial width, one directly extracts $|V_{bc}|$. In the present letter we observe, there exists a sum-rule for semi-leptonic $B$ or $\Lambda_b$ decays similar in spirit to BDT but with much weaker assumptions. In particular, the only assumption we need to make to derive our sum rule is that the $\overline{c}c$ content of the $B$ meson or $\Lambda_b$ can be neglected. If we make a second assumption, that our sum rule is saturated in the kinematically allowable region, we have a model independent extraction $|V_{\rm bc}|$ from experimental measurements without relying on heavy quark symmetry.

Underlying both the Nussinov-Wetzel-Isgur-Wise (NWIW) method and the BDT method

are the following assumptions about hadronic states which carry the quantum numbers of a heavy quark:

i) The entire heavy quark content of the state is given by a single valence heavy quark;

ii) the energy and momentum of the state is carried predominantly by the heavy quark—*i.e.* that carried by the light (anti)quarks and gluons are neglible; and

iii) the $b$ quark and $c$ quark may be considered heavy.

The meaning of assumption i) is that one may neglect virtual $\overline{c}c$ pairs and $\overline{b}b$ pairs in analyzing the state. All three of these assumptions are automatically satisfied in QCD in the formal limit of $m_b, m_c \to \infty$. The consequences of these three assumptions are profound—they imply the existence of a symmetry for such states: heavy quark symmetry. The key physical point is that if these assumptions are true, the valence heavy quark acts like a static coulomb color source in the rest frame of the hadron. Thus, in this limit, the light degrees of freedom "sees" the same color source, independent of the flavor or spin state of the heavy quark.

The essential point of NWIW method of extracing $|V_{bc}|$ is that the transition form factor for $B$ to $D$ semi-leptonic decays with zero velocity transfer is unity since the light degrees of freedom see the same static color source in both the $B$ and $D$ mesons and hence the "wavefunctions" are identical. A simple way to think about this is to suppose that the weak interaction converting a $b$ quark to a $c$ quark at the same velocity were to happen suddenly. The state basically does not change. It is clear, that this analysis depends on heavy quark symmetry in an essential way.

The BDT method is based on a key fact: in the heavy quark limit, the various form factors which in general are independent become related [3]. This greatly simplifies the analysis of inclusive semi-leptonic decays. In analyzing a given exclusive decay that forms the inclusive sum, the reduction in the number of independent structures in the transition form factor is exploited to obtain a simple sum rule which reproduces the spectator model. Again, it is clear that this analysis depends on the assumptions of heavy quark symmetry in an essential way.

Both of these methods depend critically on the fact that both the initial state containing a $b$ quark and the final state containing a $c$ quark satisfy the assumptions of heavy quark symmetry. While it is probably quite reasonable to assume that the $b$ quark is heavy, the $c$ quark is more problematical—after all it's mass has been estimated to be 1.3–1.7 GeV which is not that much larger than the typical hadronic scale of 1 GeV. [4] There are two possible sources of contamination of the extraction of $V_{bc}$ due to the finiteness of $m_c$. The first is that there may be nonvanishing $\overline{c}c$ contributions to the initial state and the second is that the energy and momentum of the final state may not be dominated by the valence $c$ quark. Of these concerns, the second is probably more serious. The $\overline{c}c$ contributions in the initial state might be expected to be particularly small. In the first place, the relevant mass scale in an energy denominator for virtual $\overline{c}c$ pairs is $2m_c$ rather than $m_c$; moreover, these contributions may well be dynamically small independent of the mass since they are Zweig rule violating. Accordingly, it is of some importance to see whether one can extract $|V_{\rm bc}|$ from experiment based only on the assumption that the $\overline{c}c$ content of the $B$ meson or $\Lambda_b$ is neglible but without reliance on the assumption that the energy and momentum of the final state is dominated by the $c$ quark. To make our assumption concrete: we assume that all matrix elements of normal ordered operators containing any $c$ quark creation or annihilation operators in a $B$–meson or $\Lambda_b$ can be neglected. Here we show that, at least in principle, one can make such an extraction from semi-leptonic decays. [7]

The partial width, $\Gamma_n(B \rightarrow l(l) + \overline{\nu}_l(q-l) + X_c(P_n))$, for the semi–leptonic decay of a $B$–meson into a particular charmed state $X_c$ is:

$$\frac{d^6\Gamma_n}{d^3l d^3q} = \frac{|M|^2 \delta^4(P_B - q - P_n)}{8M_B E(q_0 - E)(2\pi)^2} \int \prod_{f=1}^{n} \frac{d^3 f}{(2\pi)^3 2E_f} \tag{1}$$

where the phase–space is the usual product over all particles in the final state $X_c$ and where $|M|^2$ is the squared amplitude for the decay,

$$|M|^2 = \frac{G_F^2 |V_{bc}|^2}{2} \{8\, l^{\mu\nu}\} \langle P_B | J_\mu^{W\dagger}(0) | n \rangle \langle n | J_\nu^W(0) | P_B \rangle. \tag{2}$$

In the above, $E$ is the energy of the lepton, $q_0$ that of the lepton neutrino pair and

$$l^{\mu\nu} = l^\mu q^\nu + l^\nu q^\mu - 2l^\mu l^\nu - \frac{1}{2} q^2 g^{\mu\nu} + i\epsilon^{\mu\nu\alpha\beta} q^\alpha l^\beta, \tag{3}$$

is the familiar Dirac tensor that results, assuming the neutrino to be massless, from the summation over the spins of the lepton and neutrino. The sum over all states $X_c$ can then be written

$$\frac{d^6\Gamma}{d^3l d^3q} \equiv \sum_n \frac{d^6\Gamma_n}{d^3l d^3q} = \frac{G_F^2 |V_{bc}|^2}{2M_B E(q_0 - E)(2\pi)^2} l^{\mu\nu} B_{\mu\nu} \tag{4}$$

where $B_{\mu\nu}(q, P_B)$ is defined in analogy with deep–inelastic scattering as the tensor

$$B_{\mu\nu}(q) \equiv \sum_n \delta^4(P_B - q - P_n) \int \prod_{f=1}^{n} \frac{d^3f}{(2\pi)^3 2E_f} \langle P_B | J_\mu^{W\dagger}(0) | n \rangle \langle n | J_\nu^W(0) | P_B \rangle. \tag{5}$$

Consider now the sum over all states with fixed three–momentum $\vec{q}$,

$$\overline{B}^<_{\mu\nu}(\vec{q}) \equiv \int_0^{M_B - M_D} dq_0 B_{\mu\nu}(q) = \sum_n \int d^3x e^{-i\vec{q}\cdot\vec{x}} \langle P_B | J_\mu^{W\dagger}(0, \vec{x}) | n \rangle \langle n | J_\nu^W(0) | P_B \rangle \tag{6}$$

in which the upper limit of the integral has been evaluated in the rest frame of the $B$–meson and is the difference in the masses of the $B$ and $D$ mesons. It is the maximum energy the lepton–neutrino pair can carry–off as evaluated in this frame. Note that we have translated the operator $J_\mu^W$ and used the standard integral representation for the $\delta$–function to convert Eq. (5) into the above Fourier–transform. The superscript in our nomenclature is motivated by the fact that the definition of $\overline{B}^<_{\mu\nu}$ involves a sum over all charmed states $X_c$ with energy (in the rest of the $B$–meson) $P_n^0 < M_B$. In a complete sum over states one would also need to include states $X_c'$ such that $P_n^0 > M_B$. If however the kinematically available states saturate the summation, then

$$\overline{B}^<_{\mu\nu}(\vec{q}) \approx \overline{B}_{\mu\nu}(\vec{q}) \equiv \int d^3x e^{-i\vec{q}\cdot\vec{x}} \langle P_B | J_\mu^{W\dagger}(0, \vec{x}) J_\nu^W(0) | P_B \rangle. \tag{7}$$

It is $\overline{B}_{\mu\nu}(\vec{q})$ that obeys a sum rule which we will now derive using the mildest assumptions concerning the ground state of the $B$–meson. The issue whether Eq. (7) is itself a good approximation, *i.e.* whether the sum over states saturates, is ultimately a question that must be determined by experiment. We will return to this issue after we derive the sum–rule that $\overline{B}_{\mu\nu}(\vec{q})$ obeys.

We start by using a standard trick of many–body physics [8] to convert the equal–time product of fields in Eq. (7) into a time ordered product:

$$\overline{B}_{\mu\nu}(\vec{q}) = \lim_{t\to 0^+} \int d^3x e^{-i\vec{q}\cdot\vec{x}} \langle P_B | T(J_\mu^{W\dagger}(t,\vec{x}) J_\nu^W(0)) | P_B \rangle. \tag{8}$$

The relevant weak currents are

$$J_\nu^W(x) =: \overline{c}(x)\Gamma_\nu b(x) : \tag{9}$$

where $c(x)$ and $b(x)$ are the charm and bottom quark field operators respectively and

$$\Gamma_\nu = \gamma_\nu(1-\gamma_5). \tag{10}$$

Applying Wick's theorems to the time–ordered product one obtains that

$$\begin{aligned}
\langle P_B | T(J_\mu^{W\dagger}(t,\vec{x}) J_\nu^W(0)) | P_B \rangle &= \langle P_B | : \overline{b}_\alpha(t,\vec{x}) \Gamma_{\mu,\alpha\beta} c_\beta(t,\vec{x}) \overline{c}_{\beta'}(0) \Gamma_{\nu,\beta'\alpha'} b_{\alpha'}(0) : | P_B \rangle \\
&- \langle 0 | T(b_{\alpha'}(0) \overline{b}_\alpha(t,\vec{x})) | 0 \rangle \Gamma_{\mu,\alpha\beta} \Gamma_{\nu,\beta'\alpha'} \langle P_B | : c_\beta(t,\vec{x}) \overline{c}_{\beta'}(0) : | P_B \rangle \\
&+ \langle 0 | T(c_\beta(t,\vec{x}) \overline{c}_{\beta'}(0)) | 0 \rangle \Gamma_{\mu,\alpha\beta} \Gamma_{\nu,\beta'\alpha'} \langle P_B | : \overline{b}_\alpha(t,\vec{x}) b_{\alpha'}(0) : | P_B \rangle \\
&- \langle 0 | T(b_{\alpha'}(0) \overline{b}_\alpha(t,\vec{x})) | 0 \rangle \Gamma_{\mu,\alpha\beta} \Gamma_{\nu,\beta'\alpha'} \langle 0 | T(c_\beta(t,\vec{x}) \overline{c}_{\beta'}(0)) | 0 \rangle \langle P_B | P_B \rangle,
\end{aligned} \tag{11}$$

where $\alpha$ and $\beta$ explicitly label the components in Dirac space. The last term is of course a disconnected diagram and is ignorable for the case at hand. [9]

We now impose our one physical assumption that the $B$–meson has neglible virtual $c\overline{c}$ pairs. This then eliminates the first two terms on the r.h.s. in Eq. (11), leaving that

$$\langle P_B | T(J_\mu^{W\dagger}(t,\vec{x}) J_\nu^W(0)) | P_B \rangle \approx i S^c_{\beta\beta'}(t,\vec{x}) \Gamma_{\mu,\alpha\beta} \Gamma_{\nu,\beta'\alpha'} \langle P_B | : \overline{b}_\alpha(t,\vec{x}) b(0)_{\alpha'} : | P_B \rangle \tag{12}$$

where $S^c$ is the standard free–space causal (Feynman) propagator for a charm quark:

$$S^c(t,\vec{x}) = \int \frac{(\not{p} + m_c)}{p^2 - m_c^2 + i\epsilon} \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}. \tag{13}$$

Consider now the combination [10]

$$\overline{B}_E(\vec{q}) \equiv 2\overline{B}_{00}(\vec{q}) - \overline{B}_{\mu\mu}(\vec{q}). \tag{14}$$

From Equations (10) and (13) the Dirac algebra for $\overline{B}_E$ becomes simply $8p_0\gamma_0(1-\gamma_5)$, and

$$\overline{B}_E(\vec{q}) = \lim_{t\to 0^+} i8\int d^3x e^{-i\vec{q}\cdot\vec{x}}\langle P_B| : b^\dagger(t,\vec{x})(1-\gamma_5)b(0) : |P_B\rangle \int \frac{p_0 d^4p}{(2\pi)^4}\frac{e^{i\vec{p}\cdot\vec{x}-ip_0x_0}}{(p_0-E_p+i\epsilon)(p_0+E_p-i\epsilon)}. \quad (15)$$

Performing the integral over $p_0$ by closing on the lower–half plane (since $t\to 0^+$) one obtains that

$$\begin{aligned}\overline{B}_E(\vec{q}) &= \lim_{t\to 0^+} 4\int d^3x e^{-i\vec{q}\cdot\vec{x}}\langle P_B| : b^\dagger(t,\vec{x})(1-\gamma_5)b(0) : |P_B\rangle \int \frac{d^3\vec{p}}{(2\pi)^3}e^{i\vec{p}\cdot\vec{x}} \\ &= \lim_{t\to 0^+} 4\int d^3x e^{-i\vec{q}\cdot\vec{x}}\delta^3(\vec{x})\langle P_B| : b^\dagger(t,\vec{x})(1-\gamma_5)b(0) : |P_B\rangle \\ &= 4\langle P_B| : b^\dagger(0)(1-\gamma_5)b(0) : |P_B\rangle = 8M_B. \end{aligned} \quad (16)$$

This then is our sum rule.

Since our result Eq. (16) is a simple scalar, it is Lorentz invariant although the combination entering the definition of $\overline{B}_E(\vec{q})$ is not manifestly so. The corrections to Eq. (16) however are in general not Lorentz invariant. However the Lorentz invariance of our result also makes a comparison with its evaluation in various frames informative.

As we already mentioned, the experimental verification of Eq. (16) requires that the sum over states, Eq. (7), is saturated by the kinematically available states for $B$–decays. Itis an issue that can be addressed experimentally as to whether this assumption holds. If for fixed $\vec{q}$, the sum over states entering $B_E(\vec{q})$ is found to saturate experimentally with little incremental change as the hadronic final states approaching $M_B$, then the sum rule is presumably useful. We will conclude momentarily with how such a result can then be used to determine the CKM matrix element, $|V_{bc}|$, but first some remarks on kinematics and the relation between our result and that of Bjorken *et.al.* [3].

As is well known, the most general hadronic tensor $B_{\mu\nu}(q,P_B)$ in Eq. (5) can be expanded in terms of six scalar functions:

$$\begin{aligned} B_{\mu\nu}(q,P_B) &= B_1(q,P_B)P_{B\mu}P_{B\nu} + B_2(q,P_B)M_B^2 g_{\mu\nu} + B_3(q,P_B)(P_{B\mu}q_\nu + q_\mu P_{B\nu}) \\ &+ B_4(q,P_B)(P_{B\mu}q_\nu - q_\mu P_{B\nu}) + B_5(q,P_B)q_\mu q_\nu + B_6(q,P_B)\epsilon_{\mu\nu\alpha\beta}P_{B\alpha}q_\beta. \end{aligned} \quad (17)$$

Note that due to the structure of the lepton's tensor, $l^{\mu\nu}$, $B_4(q,P_B)$ never contributes tothe decay of the $B$–meson. It would thus be disastrous if our sum rule required knowledge of this function. Fortunately it does not, as the combination

$$\begin{aligned}2B_{00}(q,P_B) - B_{\mu\mu}(q,P_B) = M_B^2(B_1(q,P_B) - B_2(q,P_B)) \\ + 2M_B q_0 B_3(q,P_B) + B_5(q,P_B)(2q_0^2 - q^2).\end{aligned} \tag{18}$$

We see therefore that our sum rule requires the determination (in a Rosenbluth–like fashion) of four functions for each decay. Also note from the contraction with $l^{\mu\nu}$, the contributions of $B_3(q,P_B)$ and $B_5(q,P_B)$ to a decay rate go like $m_l^2$ and $m_l^4$ respectively, where $m_l$ is the mass of the outgoing lepton. Hence we expect that muonic decays will minimally be needed for determining $B_3(q,P_B)$, while $\tau$ decay modes will most likely be needed for $B_5(q,P_B)$.

The sum–rule of BDT is a special case of ours when using the "spectator model" assumptions that (i) the hadronic matrix elements are simply proportional to the free $b \to c$ quark transition, and (ii) that the momentum of the hadronic final–state, $P_n$, is dominated by that of the charm quark. With these two assumption, the hadronic matrix element $B_{\mu\nu}(q,P_B)$ is then given by:

$$B_{\mu\nu}(q,P_B) = \frac{2M_B F^2(q,P_B)}{E_b E_c}\left\{2P_{B\mu}P_{B\nu} - (P_{B\mu}q_\nu + P_{B\nu}q_\mu) - g_{\mu\nu}(M_B^2 - P_B\cdot q) - i\epsilon_{\mu\nu\alpha\beta}P_{B\alpha}q_\beta\right\} \tag{19}$$

where $F^2$ is a single structure function, $E_b$ and $E_c$ are the energies of the $b$ ($P_B$) and $c$ ($P_n$) quarks respectively and are included so that $F^2$ matches smoothly onto the zero–momentum recoil limit of heavy–quark symmetry [1], [2], $F^2(\vec{q}=0) = 1$. Note that $B_{\mu\nu}(q,P_B)$ involves a sum over the spins of the charm quark and an average over that of the bottom. Matching now the terms of Eq. (18) and Eq. (19) one obtains that

$$2B_{00}(q,P_B) - B_{\mu\mu}(q,P_B) = \frac{8M_B F^2(q,P_B)}{E_b E_c}(M_B^2 - P_B\cdot q) = 8M_B F^2. \tag{20}$$

Inserting Eq. (20) into the sum rule, Eq. (16), one obtains the result of BDT:

$$\int d\,q_0 F^2 = 1. \tag{21}$$

The connection of this sum rule to the NWIW method is equally obvious. If all of the assumptions underlying heavy quark symmetry are satisfied then our sum rule, when evaluated for $\vec{q} = 0$ (in the $B$ meson rest frame) will be saturated by two states, the $D$ and $D^*$. We should stress, however, that the sum-rule methods of BDT and the present paper have one advantage over the NWIW approach. In NWIW, $|V_{\rm bc}|$ is determined from a single kinematical point; this does not allow one to test the validity of the assumptions underlying the method. In contrast, the sum-rule methods allow for independent extractions of $|V_{\rm bc}|$ at any $\vec{q}$ (for which the sum rule saturates). Thus, the consistency of the extracted $|V_{\rm bc}|$ for various values of $\vec{q}$ serves as a test of the underlying assumptions.

It is also worth noting that interesting physics can be extracted from the sum rule, independent of the extraction of $|V_{\rm bc}|$. Comparisons of $|V_{\rm bc}|$ extracted by the present method and via NWIW and BDT methods gives a test of the validity of the assumptions of heavy quark symmetry which is certainly interesting in its own right. Moreover, the weaker assumption of the present sum rule, that the virtual $\overline{c}c$ pairs are neglible in a $B$ meson, can also be tested by checking the consistency of the extracted $|V_{\rm bc}|$ for various values of $\vec{q}$.

The potential usefulness of this sum-rule to extract $|V_{\rm bc}|$ should be clear. In order to make such an extraction one must measure the energy and momentum of both the final lepton and of *all* of the final hadrons. Simple kinematics allows one to do a Rosenbluth-like seperation which experimentally gives $|V_{\rm bc}|^2 B_{\mu\nu}(q_0, \vec{q})$. Implementing the sum rule allows an extraction of $|V_{\rm bc}|$. As noted above this extraction may not be easy technically since the extraction of $B_{\mu\nu}(q_0, \vec{q})$ contains pieces proportional to the square of the lepton mass, presumably requiring studies in which the final lepton is a $\tau$. Whether or not such an extraction is possible at machines available either presently or in the near future requires further study. Clearly, this method is experimentally more difficult than either the NWIW or BDT methods. On the other hand, the theoretical uncertainities associated with the extracted $|V_{\rm bc}|$ will be greatly minimized.

This work was supported in part by the U.S. Department of Energy under grant No. DE-FG05-93ER-40762 and NSF grant PHY-9058487.

## REFERENCES


[1] S. Nussinov and W. Wetzel, Phys. Rev. D **36**, 130 (1987).

[2] N. Isgur and M. Wise, Phys. Lett. B**232**, 113 (1989); *ibid*, **237**, 527 (1990).

[3] J. D. Bjorken, I. Dunietz and J. Taron, Nucl. Phys. B**371**, 111 (1992).

[4] See also [5] and [6] for other cases that large violations to naive predictions using heavy quark symmetry are expected in the charm system.

[5] T. D. Cohen and J. Milana, Phys. Lett. B**303**, 134 (1993).

[6] C. E. Carlson and J. Milana, U. of MD PP #94–062 and WM–93–110 (1993).

[7] A method for extracting CKM matrix elements from such decays complementary to the one discussed here was proposed by J. Chay, H. Georgi, and B. Brinstein, Phys. Lett. B**247**, 399 (1990); and more recently discussed by I. I. Bigi, M. Shifman, N. G. Uraltsev and A. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993); and A. V. Manohar and M. B. Wise, UCSD/PTH 93–14 and CALT–68–1883, (1993). In these approaches it is assumed that kinematic cuts on the observed lepton can be introduced to guarantee the validity of an operator product expansion. Differences between the approaches can be appreciated by noting that the zero–recoil point of NWIW, included here and in BDT, are thus excluded by design.

[8] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), Sec. 17.

[9] Strictly, we have a sum–rule for $\langle P_B|J_\mu^{W\dagger}(0,\vec{x})J_\nu^W(0)|P_B\rangle - \langle 0|J_\mu^{W\dagger}(0,\vec{x})J_\nu^W(0)|0\rangle$ where $|0\rangle$ is the perturbative vacuum. We note however that in the kinematically accessible region, there is no contribution from the perturbative piece.

[10] This unusual combination was chosen since it leads to a simple sum–rule. We note that an analogous combination is useful in deep inelastic scattering, see T. D. Cohen, D. K.

Griegel, U. of MD PP #94–091.